Period 1, May 28, 2025

2. $\dfrac{5x+7}{x^3+2x^2-x-2} = \dfrac{5x+7}{(x+2)(x+1)(x-1)} = \dfrac{A}{(x+2)} + \dfrac{B}{(x+1)} + \dfrac{C}{(x-1)}$

$x^2(x+2) - 1(x+2)$

$(x+2)(x^2-1)$

$(x+2)(x+1)(x-1)$

$$\frac{5x+7}{(x+1)(x-1)(x+2)} = \frac{A(x+1)(x-1)}{(x+1)(x-1)(x+2)} + \frac{B(x+2)(x-1)}{(x+1)(x-1)(x+2)} + \frac{C(x+2)(x+1)}{(x+1)(x-1)(x+2)}$$

$5x+7 = A(x^2-1) + B(x^2+x-2) + C(x^2+3x+2)$

$5x+7 = Ax^2 - A + Bx^2 + Bx - 2B + Cx^2 + 3Cx + 2C$

$0x^2 = Ax^2 + Bx^2 + Cx^2$

$5x = Bx + 3Cx$

$7 = -A - 2B + 2C$

$0 = A + B + C$

$5 = B + 3C$

$7 = -A - 2B + 2C$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 5 \\ -1 & -2 & 2 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & 3 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 6 & 12 \\ 0 & -1 & 3 & 7 \end{array}\right]$$

$6C = 12$

$C = 2$

$-B + 3C = 7$

$-B + 3(2) = 7$

$-B + 6 = 7$

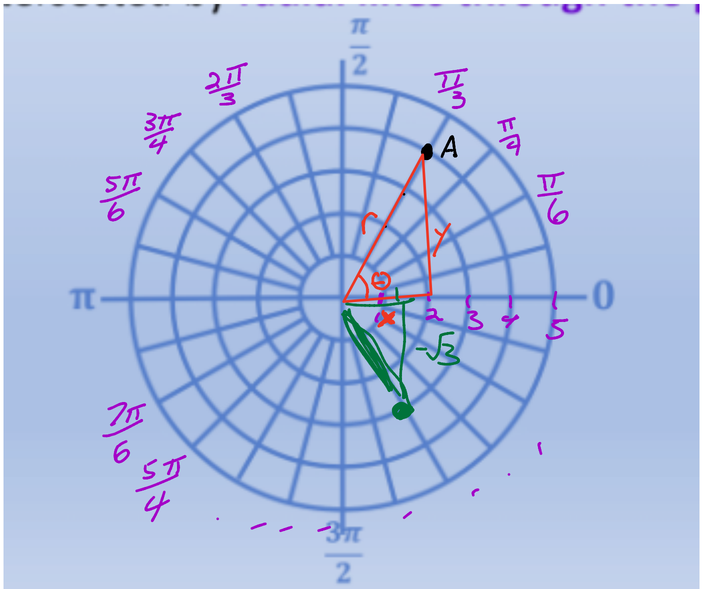
$B = -1$

$A + B + C = 0$

$A - 1 + 2 = 0$

$A = -1$

$$\frac{5x+7}{x^3+2x^2-x-2} = \frac{-1}{(x+2)} + \frac{-1}{(x+1)} + \frac{2}{(x-1)}$$



$A \quad \left(4, \frac{\pi}{3}\right), \left(4, \frac{7\pi}{3}\right), \left(-4, \frac{4\pi}{3}\right)$

$r \cdot \cos\theta = \frac{x}{r} \cdot r \quad \Big\} \quad r\cos\theta = x$

$r \cdot \sin\theta = \frac{y}{r} \cdot r \quad \Big\} \quad r\sin\theta = y$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{\cancel{r}} \cdot \frac{\cancel{r}}{x} = \frac{y}{x}$

$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

Ex. Change from polar to rectangular

$(-4, \pi)$ $\quad x = -4\cos\pi = -4 \cdot -1 = 4$

$y = -4\sin\pi = -4 \cdot 0 = 0$

$(4, 0)$

$\left(\sqrt{2}, \frac{\pi}{4}\right)$

Ex. Change from rectangular to polar

$(1, -\sqrt{3})$ $\quad r = \sqrt{x^2 + y^2}$

$r = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = 2$

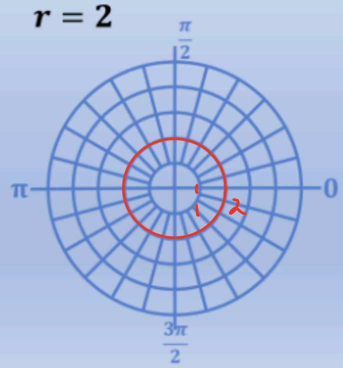
$\tan\theta = \frac{-\sqrt{3}}{1} \rightarrow -\frac{\pi}{3} = \theta$

$\left(2, -\frac{\pi}{3}\right), \left(2, \frac{5\pi}{3}\right)$

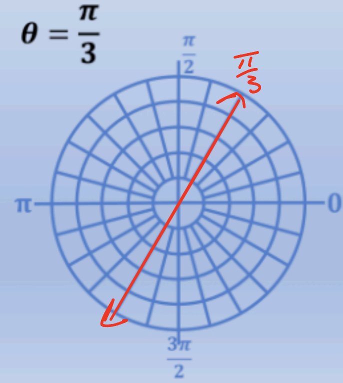
$(0, -2)$

Graphing Polar Equations

$r = 2$



$\theta = \frac{\pi}{3}$

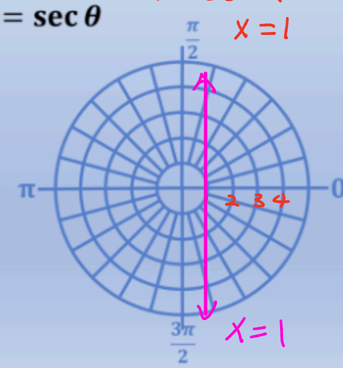


$r = \sec\theta$

$r = \frac{1}{\cos\theta}$

$r\cos\theta = 1$

$x = 1$

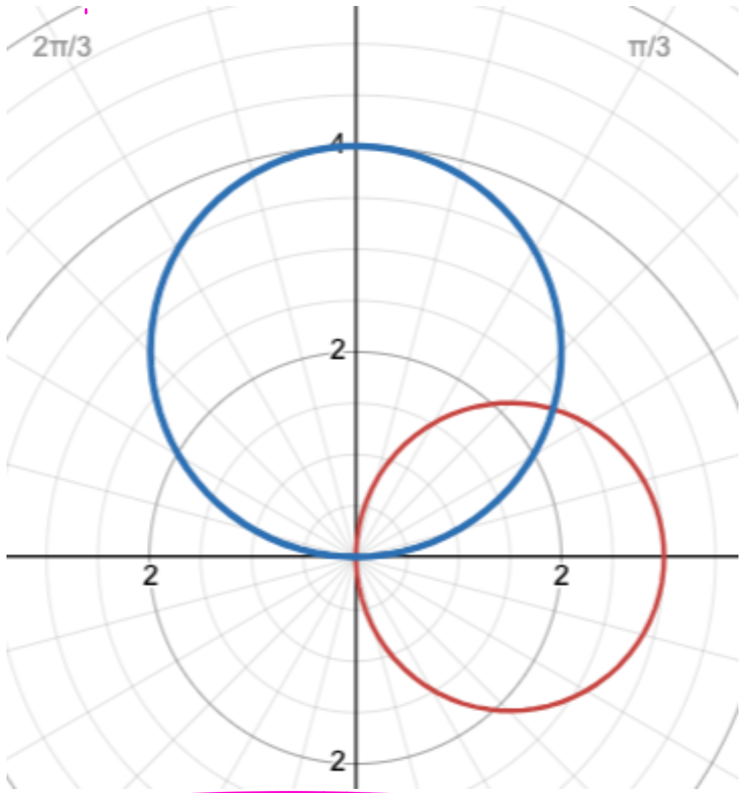


$r = 3\cos\theta$

$0 \le \theta \le \pi$

$r = 4\sin\theta$

$0 \le \theta \le \pi$



$y = r\sin\theta$

$x = r\cos\theta$

$\frac{dy}{dx}$

$r = f(\theta)$

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{dy}{dx}$$

$y = F(\theta)\sin\theta$

$x = F(\theta)\cos\theta$

$$\frac{dy}{d\theta} = F'(\theta)\sin\theta + F(\theta)\cdot\cos\theta$$

$$\frac{dx}{d\theta} = F'(\theta)\cos\theta + F(\theta)(-\sin\theta)$$

$$\text{Slope} = \frac{dy}{dx} = \frac{F'(\theta)\sin\theta + F(\theta)\cos\theta}{F'(\theta)\cos\theta) - F(\theta)(\sin\theta)}$$

Find the polar coordinates of the point(s) of intersection of the given curves for $0 \le \theta < 2\pi$.

15) $r = 6$, $r = 2 + 4\sin\theta$

$6 = \cancel{2} + 4\sin\theta$

$-2 \quad -2$

$4 = 4\sin\theta$

$1 = \sin\theta$

$? = \theta$

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13) $r - 3\cos\theta = 8\sin\theta$

$+3\cos\theta \quad +3\cos\theta$

$r \cdot r = (8\sin\theta + 3\cos\theta)r$

$r^2 = 8r\sin\theta + 3r\cos\theta$

$x^2 + y^2 = 8y + 3x$

$x^2 - 3x + \frac{9}{4} + y^2 - 8y + 16 = 0 + \frac{9}{4} + 16$

$a = 1$
$b = -3$
$\frac{b}{2} = \frac{-3}{2}$
$\left(\frac{b}{2}\right)^2 = \frac{9}{4}$

$a = 1$
$b = -8$
$\frac{b}{2} = \frac{-8}{2} = -4$
$\left(\frac{b}{2}\right)^2 = 16$

$(x - \frac{3}{2})^2 + (y - 4)^2 = \frac{9}{4} + \frac{64}{4}$

$(x - \frac{3}{2})^2 + (y - 4)^2 = \frac{73}{4} \Rightarrow$ Circle center $(\frac{3}{2}, 4)$

$r = \frac{\sqrt{73}}{2}$